TECHNICAL MEMORANDUM 805

FEBRUARY 1968

LOW DENSITY TRANSITIONAL REGIME DRAG COEFFICIENTS FOR SLENDER COLD WALL CONICAL VEHICLES IN HYPERSONIC FLOW (U)

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A PROJECT DEFENDER TASK
Sponsored by the
ADVANCED RESEARCH PROJECTS AGENCY
Department of Defense
Washington, D.C. 20301

This task was conducted under the sponsorship of the Advanced Research Projects Agency (ARPA Order 958-67) and monitored by the U.S. Army Missile Command, AMSMI-RNS, under Contract DA-AH01-67-C-1334.

ABSTRACT

A survey of numerical results of low-density transitional regime drag coefficients and number density distributions about slender, cold-wall, conical vehicles at hypersonic speeds is presented. The transitional regime results have been obtained from numerical solutions of Rosen's restricted variational principle representation of the Boltzmann equation, which has been shown to be equivalent to Galerkin's method of solution of operator equations. Transitional regime drag coefficients are compared with near free molecule and viscous interaction predictions of drag coefficients. Number density profiles for the transitional regime are also given and compared with known qualitative features of near continuum flow fields to assess the validity of results obtained by the variational method.

Results indicate that the drag coefficient varies smoothly from known viscous interaction predictions and reaches the limiting value of 2 when the freestream Knudsen number is on the order of unity for the class of slender cones considered. A decrease of cone half-angle, with Knudsen number fixed, results in a steeper rise of the drag coefficient curve toward the limiting high Knudsen number value of 2. For cone half-angles on the order of 5 degrees, the drag coefficient curve exhibits an overshoot over the free molecule value.

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I. INTRODUCTION

In Refs. 1 and 2, Rosen's[3] restricted variational principle method of solving the Boltzmann equation was used to determine the hypersonic low-density transitional regime flow over conical bodies. Number density profiles as well as drag coefficients were calculated for Knudsen numbers* in the transitional regime between 10^{-2} and about 10^{1} for the case of an adiabatic wall, and the results compared with those yielded by inviscid continuum theory,[4] free molecule theory,[5] the near-free molecule predictions of Laurmann,[6] the viscous interaction theory of Ellinwood and Mirels,[7] and the experimental results of Hickman.[8]

The present note presents further results obtained by using this variational method to determine number density profiles and drag coefficients in the Kn_∞ regime between 10^{-3} and 10^{2}, under typical reentry conditions of hypersonic flow over cold-walled, slender cones. Since Refs. 1 and 2 fully present the theoretical aspects of the variational method, only numerical results are given in this note. The evolution of number density profiles with decreasing Knudsen number is followed down to low Knudsen numbers ($Kn_\infty = 10^{-3}$) to determine whether the variational method is capable of reproducing certain well-known qualitative details of continuum flow fields. Drag coefficient and number density distribution results are given for 5- and 10-degree cones to study the effects of body geometry and Knudsen number on low-density flow fields, and qualitative trends of these results are discussed.

*Abbreviated Kn_∞ and defined as the free stream mean free path normalized to body length.

II. RESULTS

Figure 1 represents a composite of C_D predictions for the theories given in Refs. 1-7 for a $M_\infty = 16$ flow over a cold wall 10-degree half-angle cone in the Knudsen number range between 10^{-5} and 10^{2} (where M_∞ is the free stream Mach number). The viscous interaction predictions of Ellinwood and Mirels[7] overlap the C_D predictions given by the variational method for Kn_∞ in the range between 10^{-2} and 3×10^{-2}. Further, the variational method results qualitatively agree with the trend of Laurmann's[6] near free molecule C_D variation for Kn_∞ greater than 1. The C_D variation predicted by the variational method provides a smooth transition between the limits of applicability of these two theories. Figure 2, which gives a summary of C_D predictions for the 5-degree half-angle cone case, indicates that C_D increases more rapidly for the 5-degree cone for Kn_∞ greater than about 10^{-3} than for the 10-degree cone. Further, a small C_D overshoot over the free molecular C_D value is evident both from the near free molecule and variational theories, although the overshoot is less than might be expected by the trend of the near free molecular theory. Toward the lower Knudsen number end of the curve, the region of overlap of variational and viscous interaction theories appears to begin for Kn_∞ below about 10^{-2}. For both the 5- and 10-degree cones, C_D is close to the free molecular limiting value for Kn_∞ of about 1.

Figures 3, 4, and 5 represent the evolution of number density profiles with increasing Knudsen number for the 10-degree cone and give number density profiles for $Kn_\infty = 10^{-2}$, 10^{-1}, and 1, respectively. As Kn_∞ decreases the disturbance region appears to narrow (compare Fig. 3 to Fig. 5) in the vertical direction. For higher Kn_∞, for instance Kn_∞ greater than 10^{-1}, the sensible disturbance region appears confined to distances less than a free stream mean free path;* only for Kn_∞ below 10^{-1} do the disturbance regions appear to exceed one free stream mean

*The mean free path referred to here is based on free-stream conditions; within the disturbed flow region the mean free path is less.

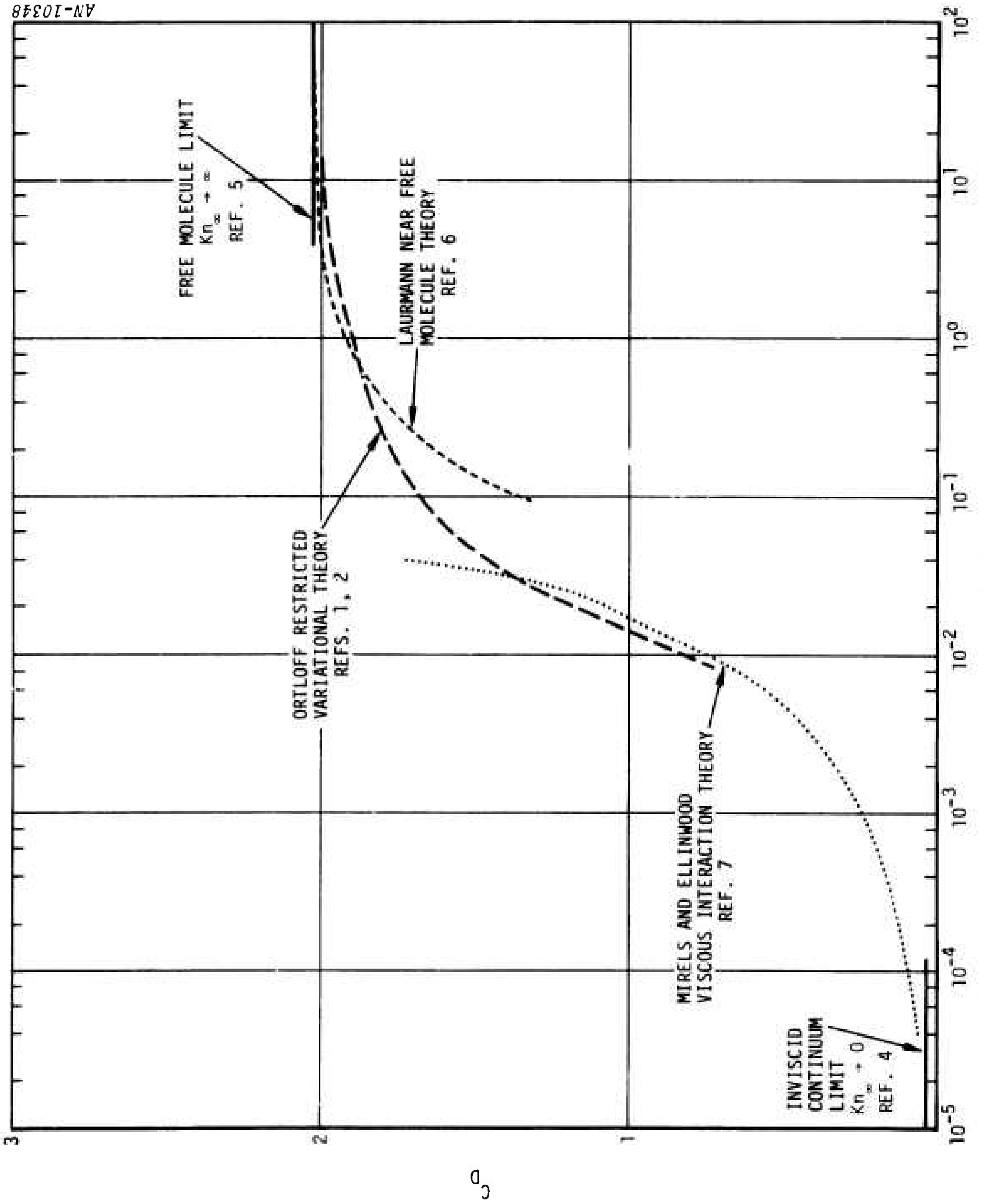

Figure 1. Drag Coefficient as a Function of Kn_∞, ψ_b = 10 degrees, M_∞ = 16, cold wall conditions

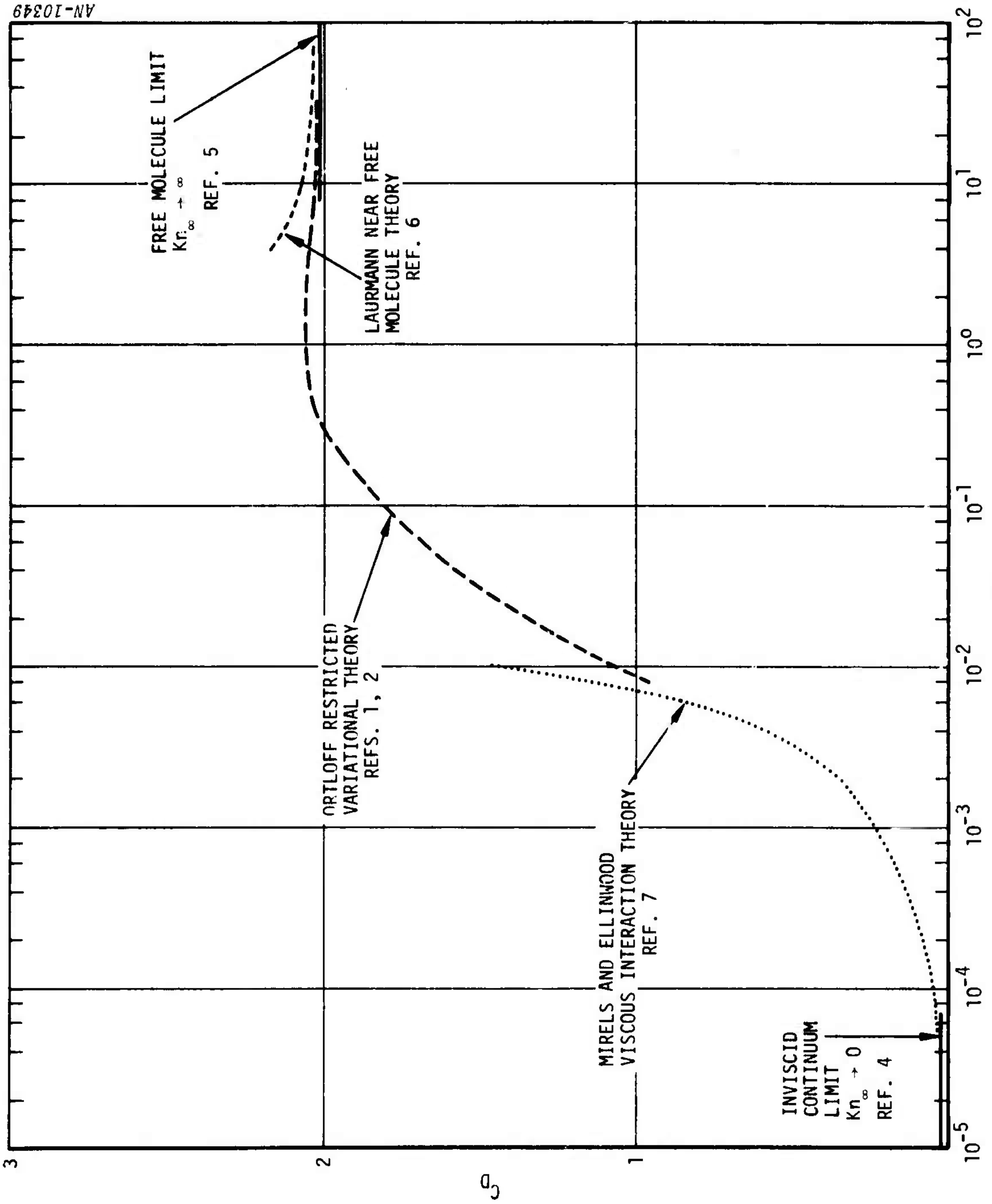

Figure 2. Drag Coefficient as a Function of Kn_∞, ψ_b = 5 degrees, M_∞ = 16, cold wall conditions

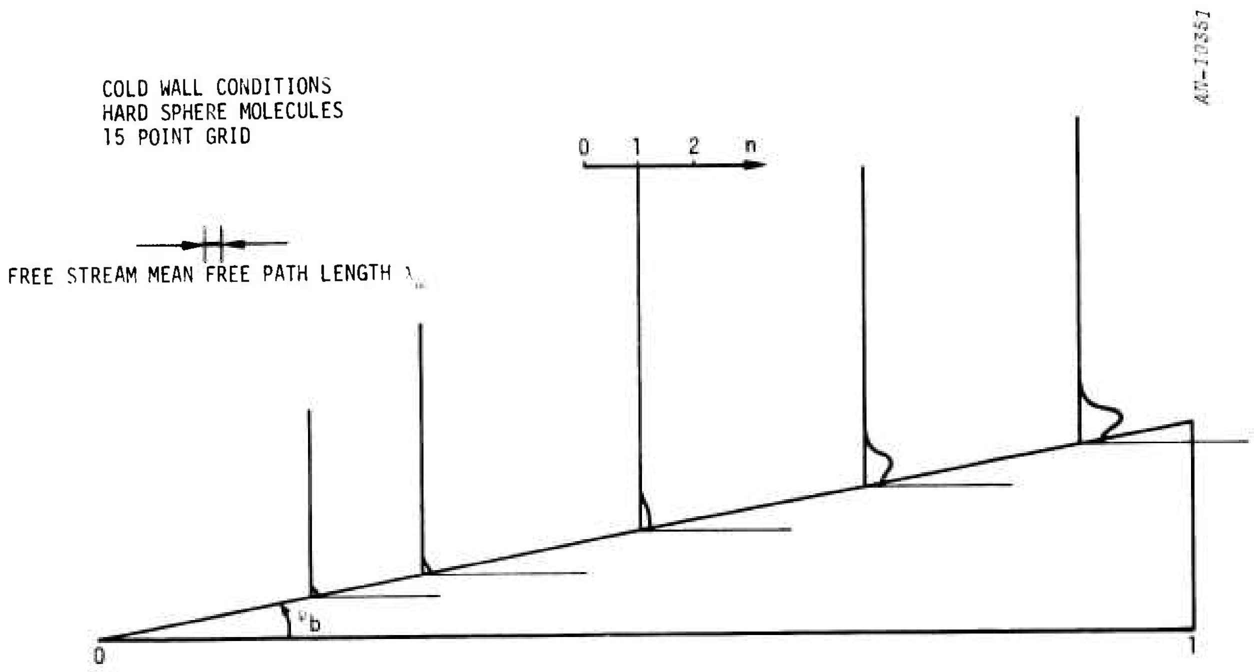

Figure 3. Number Density Profiles $M_\infty = 16$, $\psi_b = 10°$, $Kn_\infty = 0.01$

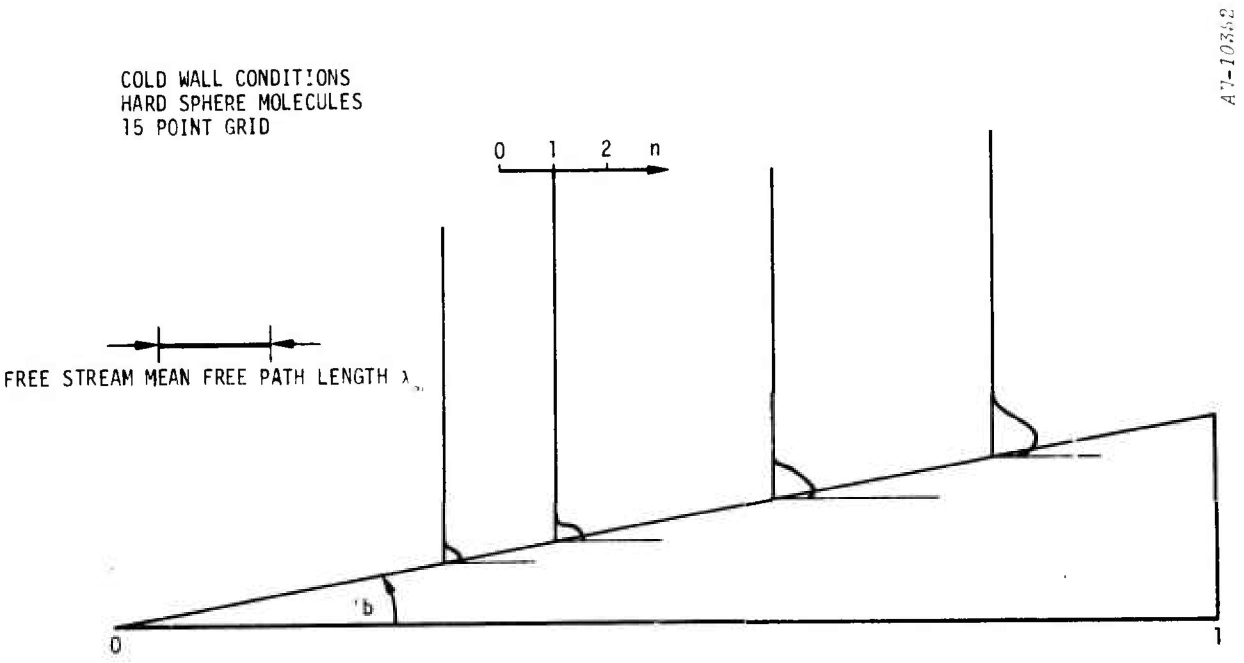

Figure 4. Number Density Profiles $M_\infty = 16$, $\psi_b = 10°$, $Kn_\infty = 0.10$

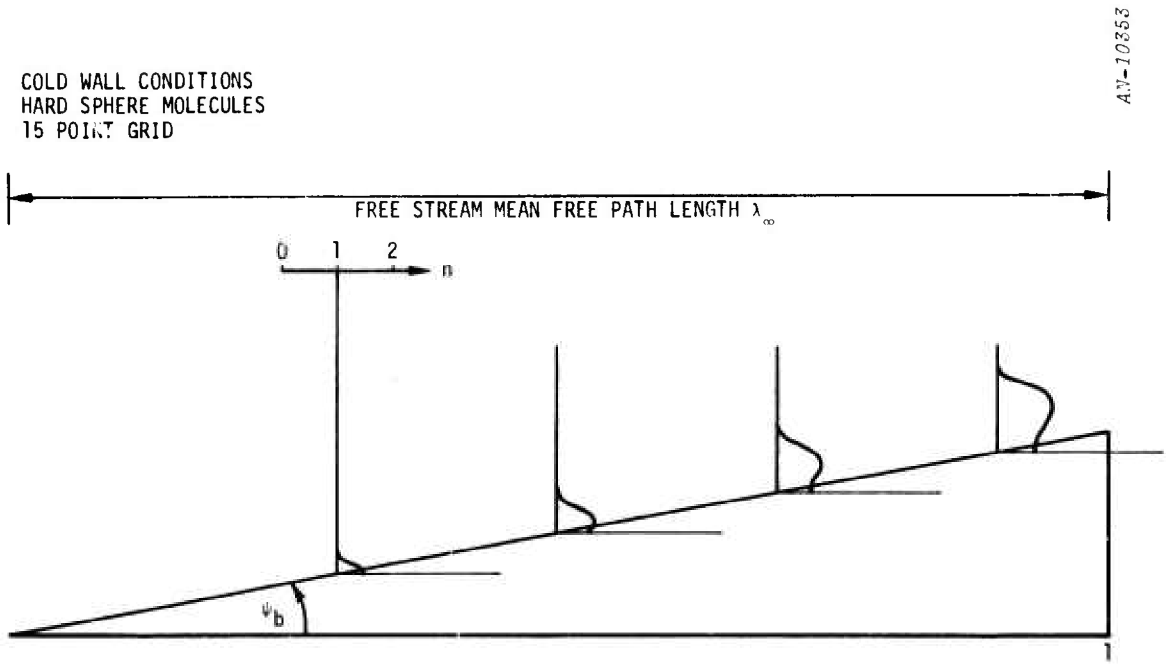

Figure 5. Number Density Profiles $M_\infty = 16$, $\psi_b = 10°$, $Kn_\infty = 1.0$

free path in vertical extent. The appearance of a number density maximum in the flow adjacent to the cone is apparent. For Kn_∞ equal to 10^{-1} and 10^{-2} the maxima are at roughly the same height above the cone surface, although for Kn_∞ equal to 10^{-2} the increasing importance of gas particle collisions appears to cause a decrease in the particle population. Although the vertical height of the number density maxima appears to be the same, the number of mean free paths away from the cone surface of these maxima continually increases as Kn_∞ decreases. In all cases, the appearance of number density maximums occurs towards the rear of the cone.

Figures 6, 7, 8, and 9 are number density profiles for $Kn_\infty = 10^{-3}$, 10^{-2}, 10 and 10^2, respectively, and represent the evolution of number density profiles for a 5-degree half-angle cone. For Fig. 6, the Knudsen number is 10^{-3}, which is close enough to continuum flow conditions ($Kn_\infty \to 0$) that qualitative features of the continuum shock layer should be apparent if the variational theory is valid to such low Kn_∞ values.

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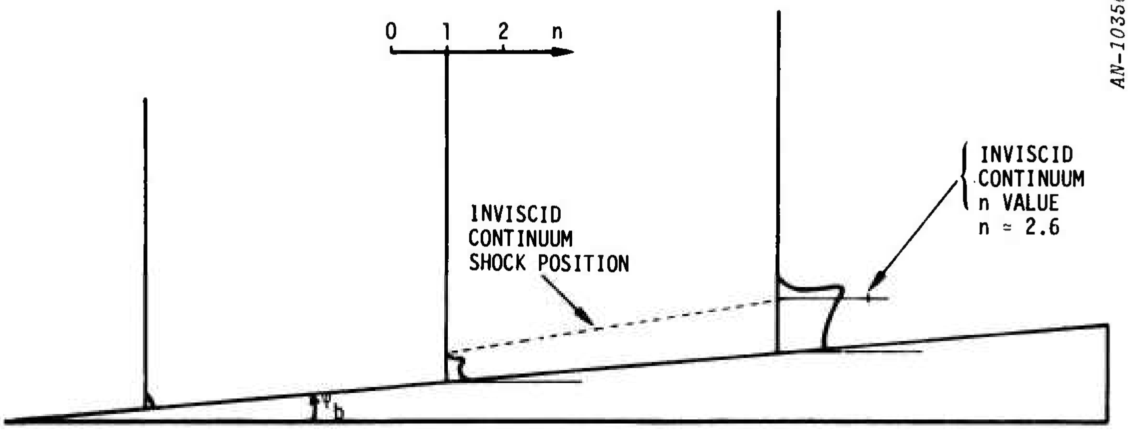

Figure 6. Number Density Profiles $M_\infty = 16$, $\psi_b = 5°$, $Kn_\infty = 0.001$

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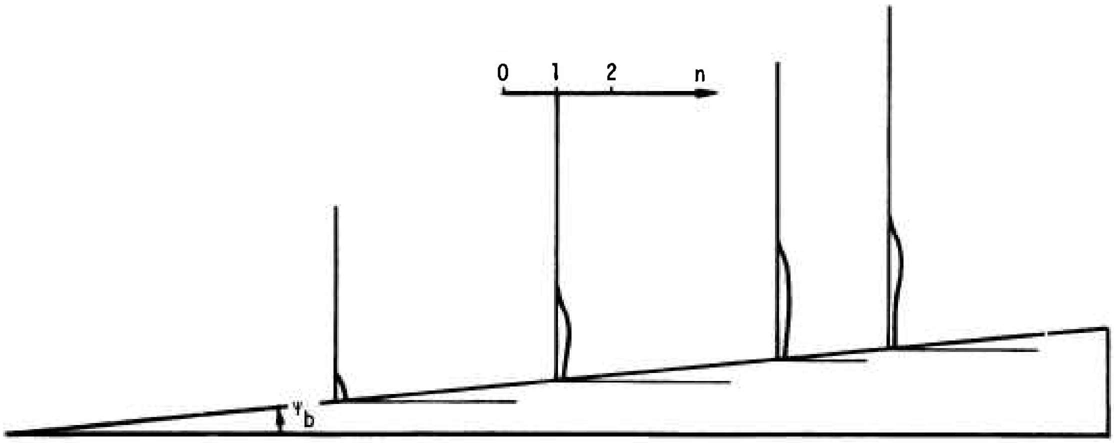

Figure 7. Number Density Profiles $M_\infty = 16$, $\psi_b = 5°$, $Kn_\infty = 0.01$

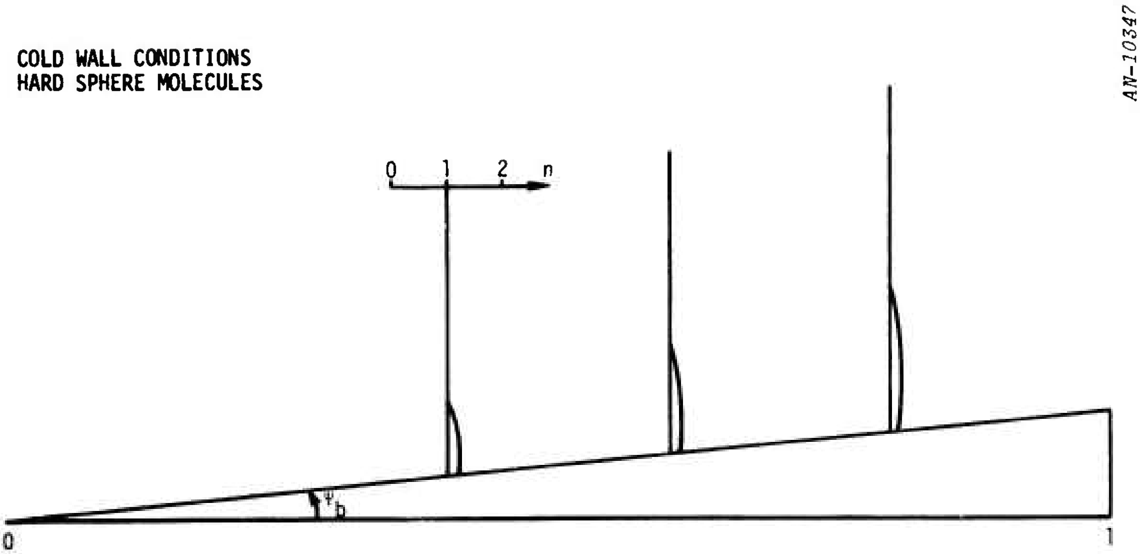

Figure 8. Number Density Profiles $M_\infty = 16$, $\psi_b = 5°$, $Kn_\infty = 10$

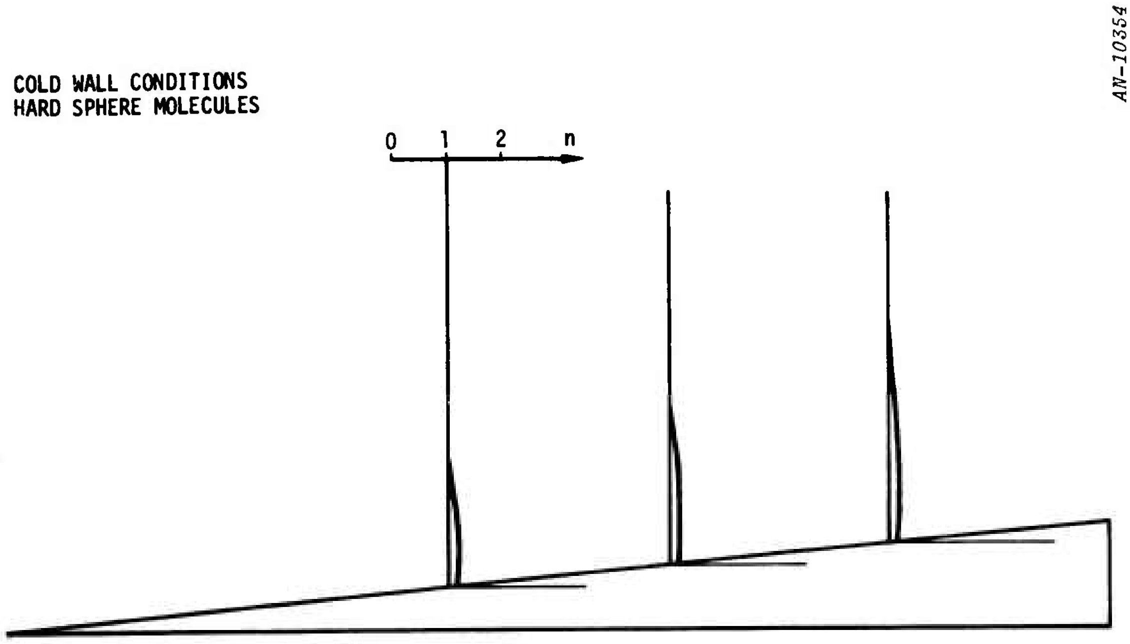

Figure 9. Number Density Profiles $M_\infty = 16$, $\psi_b = 5°$, $Kn_\infty = 100$

The results show that the diffusive number density profiles characteristic of higher Kn_∞ values (Fig. 7 for example) develop an increasingly sharp gradient with decreasing Kn_∞ (Fig. 6), typical of a fully developed shock layer. The amplitude of the number density maximum in the $Kn_\infty = 10^{-3}$ case (Fig. 6) is $\eta/\eta_\infty = 2.12$; This compares favorably to the post-shock number density predicted from classical inviscid continuum theory of $\eta/\eta_\infty = 2.6$. Further, a line drawn through the number density maxima compares favorably to the inviscid continuum shock position (shown in Fig. 6). For both the 5- and 10-degree cone cases, a definite evolution occurs from the diffusive small gradient profiles characteristic of high Kn_∞. Comparison of Fig. 7 to Fig. 3 indicates that the higher cone angle yields higher maximum number density values in a narrower disturbance region than for the 5-degree cone. The latter case is characterized by more diffusive-type profiles than the 10-degree cone but still exhibits the qualitative feature of having number density maxima away from the wall. Towards the tip region, density gradients are small; larger gradients are only developed towards the rear portions of the cone for Kn_∞ below about 10^{-2} for the 5-degree cone and below about 2 for the larger 10-degree cone. In all the cases calculated, wall number density exceeds the free stream value. This is due both to the fluid compression effect which leads to an increase in wall density with increasing cone angle (compare Fig. 3 to Fig. 7 for instance), and to the effect of the cold wall, which becomes more efficient in raising the wall number density level as Kn_∞ decreases (as wall velocity slip decreases as Kn_∞ decreases; compare Fig. 9 to Fig. 6, for example).

Comparison of disturbance regions associated with the high Kn_∞ flow over slender cones (compare Figs. 6 to 9 for the 5-degree cone to Figs. 3 to 5 for the 10-degree cone) shows that larger values of number density exist further from the cone surface for the more slender cones. This effectively results in an increased probability of collisions of free stream particles with the larger cloud of particles surrounding the more slender cone. The occurrence of large number density regions away from a body surface and the consequent increase of scattered particles to the body surface for high Kn_∞ flows have been noted by Bird[9] to result in a near-free molecule C_D overshoot above the free molecule limiting value. The presence of the C_D overshoot for the 5-degree cone is consistent with the larger disturbance region for this geometry compared to the 10-degree cone case, for which no C_D overshoot exists.

A further comparison of the free-stream Mach number effect on the extent of the number-density disturbance region is obtained by comparison of Fig. 10 to Figs. 4 and 5. In Fig. 10 the Mach number is 5.4; in Figs. 4 and 5 it is 16. For the lower Mach numbers, the vertical extent of the disturbance region is on the order of 10 to 15 times that for the higher Mach number. In addition, the profiles of Fig. 10 exhibit a more diffuse character due to the greater importance of thermal motions of gas particles at lower Mach numbers.

The survey of number density distributions and drag coefficients given in this report is meant to provide details of transitional regime flow over conical vehicles at typical reentry conditions. Little theoretical or experimental work has yet been done on transitional regime flow fields over cones, although viscous interaction and free molecule theories, which give asymptotic limits that transitional regime results must agree with, are available. In comparison with these theories, the variational method yields smooth transitions of drag coefficient variation between known correct limiting cases; additionally, details of continuum and free molecule flow fields are likewise obtained from the variational method for cases of $Kn_\infty = 10^{-3}$ and $Kn_\infty = 10^{2}$, respectively.

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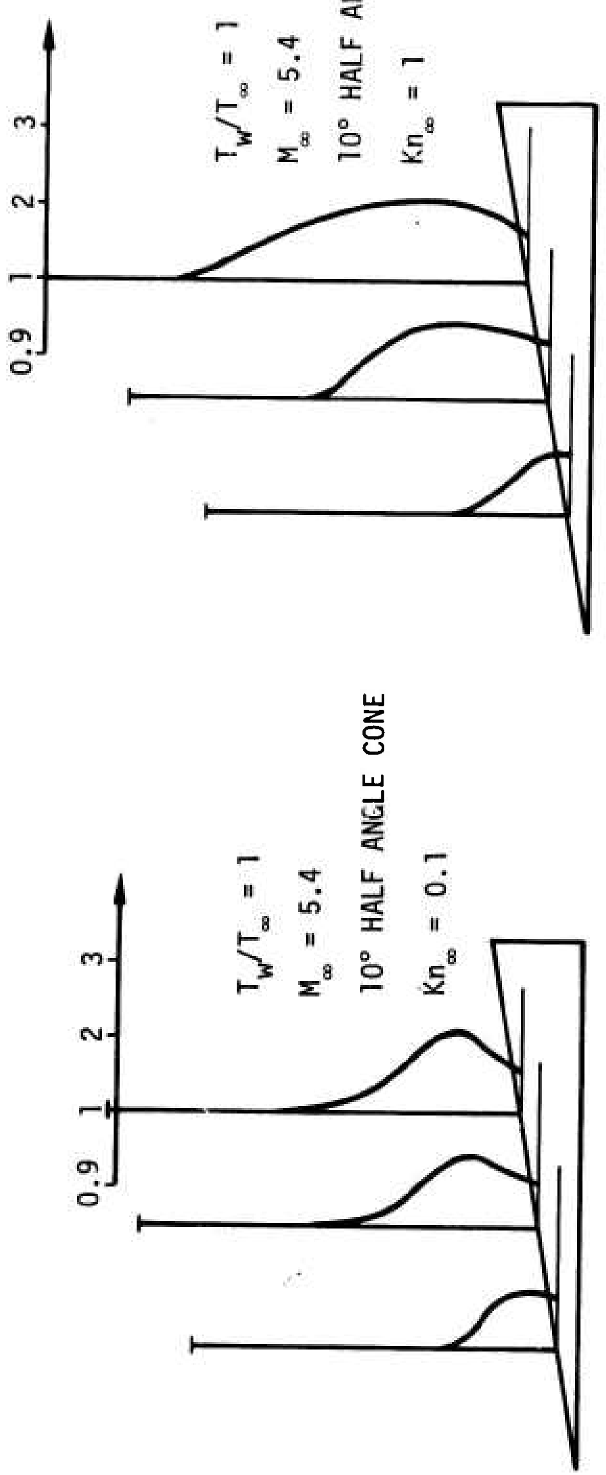

Figure 10. Number Density Distribution

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UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) GENERAL RESEARCH CORPORATION	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED
	2b. GROUP -

3. REPORT TITLE

Low Density Transitional Regime Drag Coefficients for Slender Cold Wall Conical Vehicles in Hypersonic Flow(U)

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
Technical Memorandum 805 January 1, 1967 through October 13, 1968

5. AUTHOR(S) (First name, middle initial, last name)

C. R. Ortloff

6. REPORT DATE February 1968	7a. TOTAL NO. OF PAGES 12	7b. NO. OF REFS 9
8a. CONTRACT OR GRANT NO. DA-AH01-67-C-1334 ARPA Order 958-67	9a. ORIGINATOR'S REPORT NUMBER(S) TM-805	
b. PROJECT NO.		
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
d.		

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11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Advanced Research Projects Agency The Pentagon Washington, D.C.

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DD FORM 1 NOV 65 1473

UNCLASSIFIED

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Drag Coefficients						
Hypersonic Flow						
Slender Cones						
Transitional Regime						
Variational Principle						